In search of catchment scale physics- estimating catchment scale groundwater dynamics from recession analysis and mean annual runoff

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Summary

- A formulation of the dynamics of subsurface storage, free of calibration parameters, is presented.
- The subsurface storage is parametrised using cathment scale information such as recession data (Λ) and mean annual runoff (MAR).
- No loss in precision wrt simulated runoff is found using the new routine.
- Recessions are better simulated, suggesting more realistic groundwater dynamics
- The approach inspires searching for a method for updating the subsurface storage S (and thereby Qsim) from Qobs

«The important hydrological action takes place underground» (Beven, 2001)

- There is a (quite a) gap between theoretical (Dupuit-Boussinesq, Darcy) and conceptual (operational) formulations of the hydrological subsurface.
- Countless concepts are presented for modelling the subsurface, recognizing that thats where the dynamics of runoff is formed.
- How can we close this gap and what are the «physically based equations for hydrological behaviour at the catchments scale»? (Kirchner, 2006)

DDD model

-a parameter parsimonius rainfall-runoff model (Skaugen and Onof, 2014, Skaugen et al. 2015)

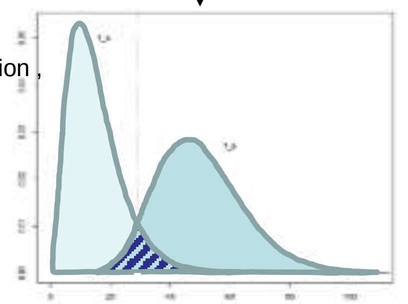
- runoff dynamics are modelled by unit hydrographs arranged in parallell, turned on and off according to level of saturation

- The paramters of the unit hydrographs are determined from observed data, no calibration

Input: precipitation and temperature
10 elevation zones.

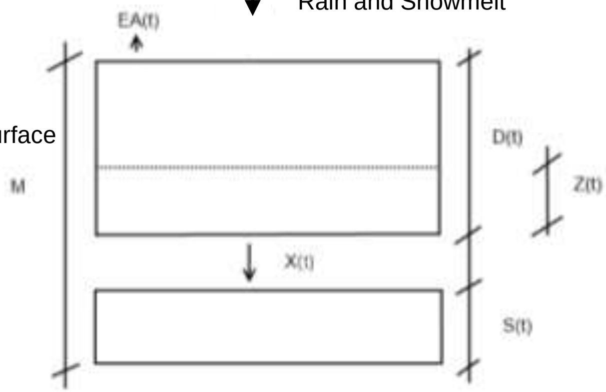
P, T,..

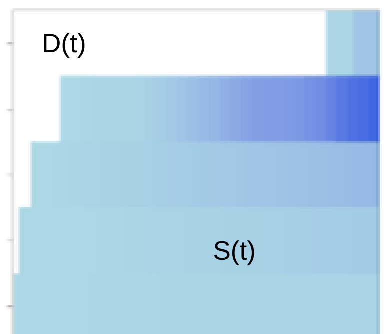
Snow distribution: accumulation , melt and snow-free area .
10 elevation zones.



Rain and Snowmelt

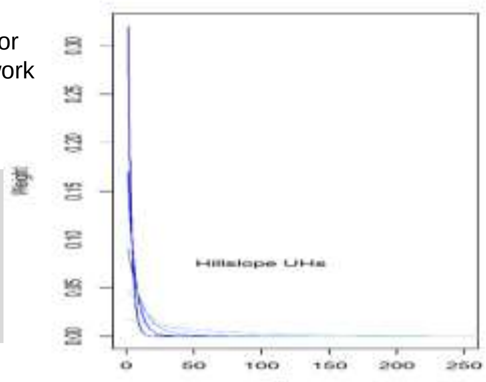
Subsurface: saturated and unsaturated zone.
Right: simulated subsurface moisture distribution in hillslope

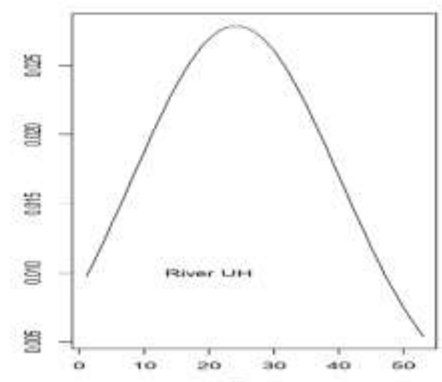




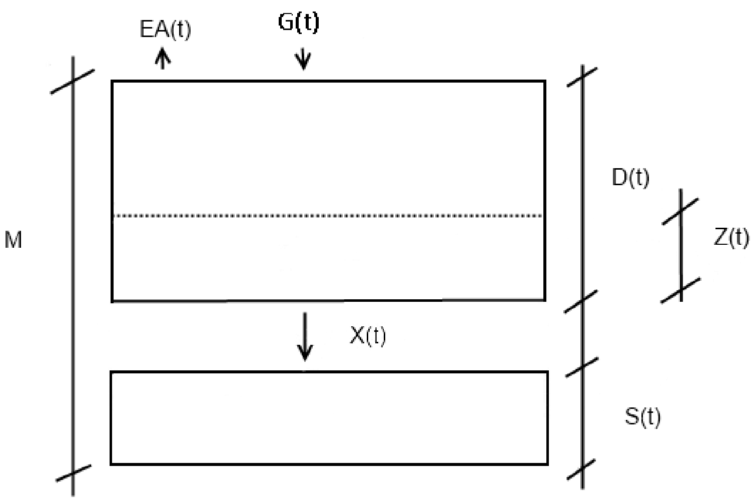
X(t) distributed in time by UHs conditioned on subsurface state, S

Runoff dynamics: UHs for hillslopes and river network





Subsurface in the DDD



EA(t)- evapotranspiration
G(t)- input rain/snowmelt
Z(t)- actual soilmoisture
D(t)- volume unsaturated zone (soilwater)
S(t)- volume saturated zone (groundwater)
X(T)- water released to S(t) and runoff

Excess water: $X(t) = Max\left\{\frac{G(t)+Z(t)}{D(t)} - R, 0\right\} D(t)$.

Groundwater: $\frac{dS}{dt} = X(t) - Q$(t).

Soil water content: $\frac{dZ}{dt} = G(t) - X(t) - Ea(t)$.

Soil water zone: $\frac{dD}{dt} = -\frac{dS}{dt}$,

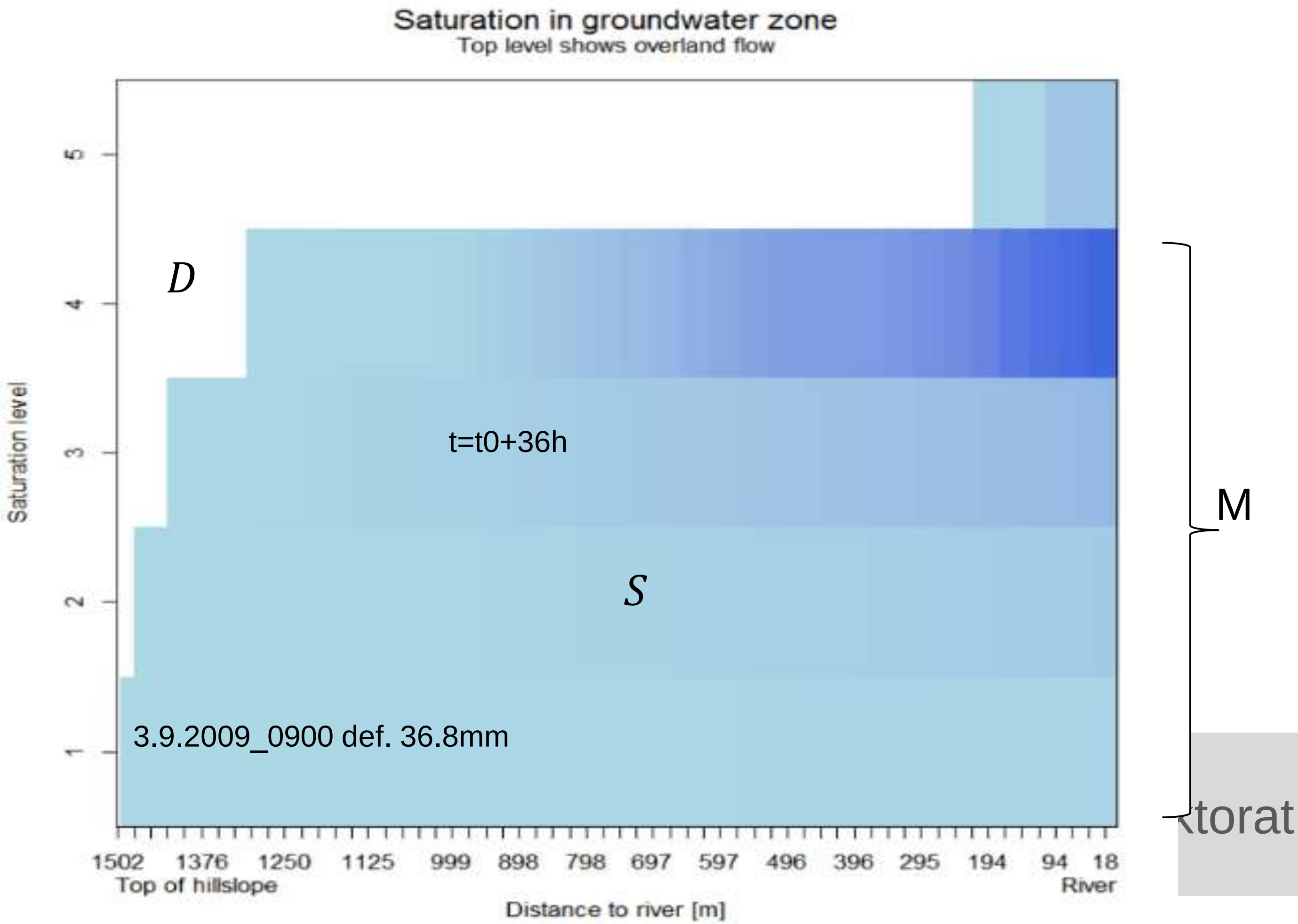
M- subsurface capacity is **calibrated!**

Q(t)- runoff
R - (field capacity: 30% of D)

Subsurface in DDD

2-D representation:

x: length of hillslope (entire catchment is represented as a hillslope)
z: moisture varying with (relative) depth



Problems of current subsurface formulation

1) M is an extreme value (hard to estimate)
2) M is a calibration parameter
3) The fluctuations of S are uniformly distributed, each level of storage is equally probable. Is that a probable model??

New formulation:

1) As in previous formulation, subsurface storage and runoff have a strong link (recession analysis)
2) We estimate the mean of the distribution of $S(t)$, m_S, less uncertainty compared to an extreme value (M).
3) We estimate m_S from data; no calibration

Recession analysis; a classic, but still underexplored source of information.

- In DDD recession, sampled from:

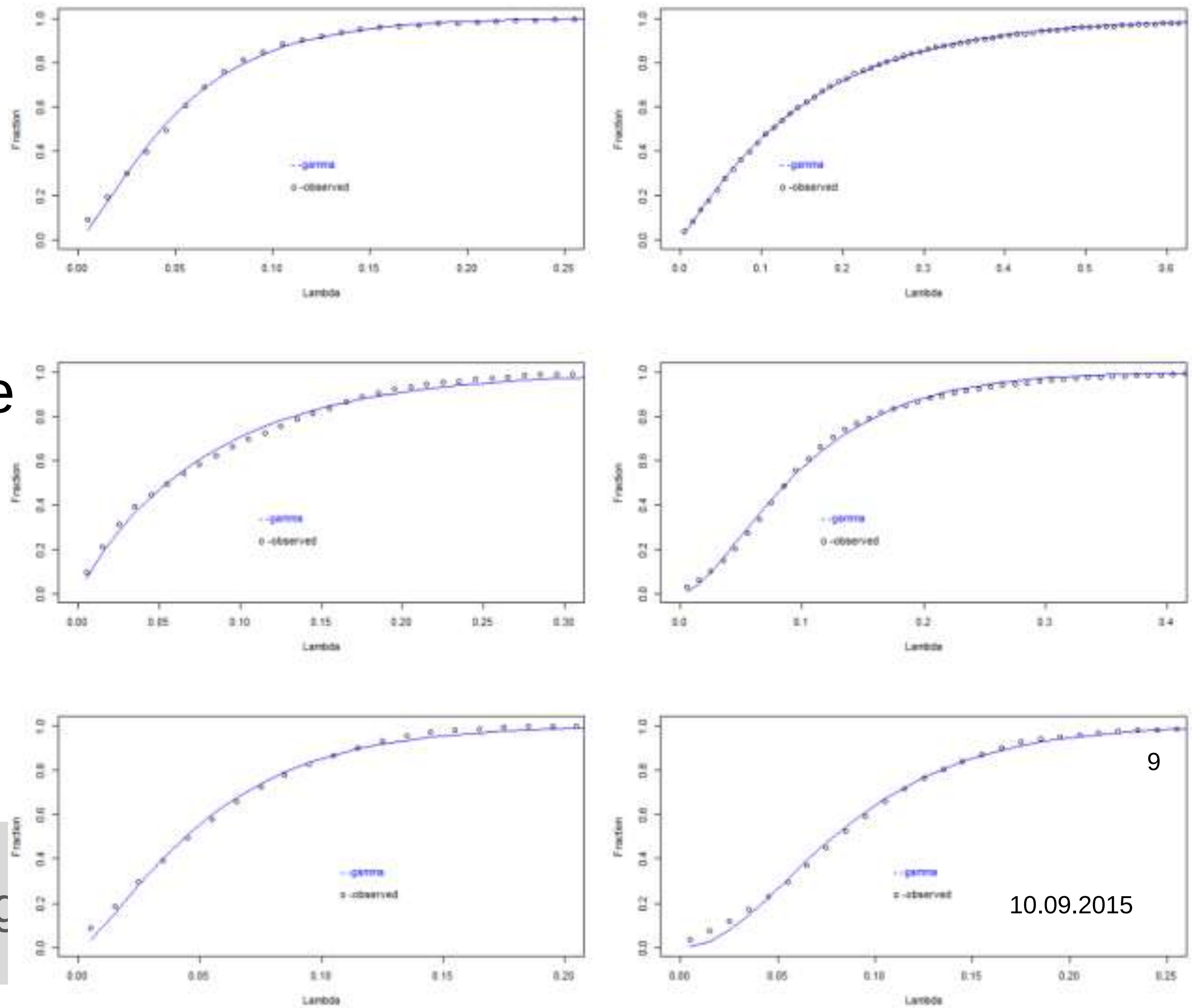
$$\Lambda = log(Q(t)) - log(Q(t + \Delta t))$$

is used to determine:

1. subsurface wave velocites, i.e assigning scale to the UHs for different levels of saturation
2. the frequency distribution of subsurface storage

A gamma distribution models the distribution of Λ

- Λ appears to be a quite robust recession characteristic. Its distribution is quite insensitive to whether we allow precipitation on day $t + \Delta t$ or not



Assumption:the distribution of S is a scaled version to that of the recession characteristic Λ

The distribution of $\Lambda = log(Q(t)) - log(Q(t + \Delta t))$ is modeled by a two parameter gamma distribution (shape and scale).

$$f(\Lambda) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)}\Lambda^{\alpha-1}\exp(-\Lambda/\beta)$$

$$f(S) = \frac{1}{\eta^{\alpha}\Gamma(\alpha)}S^{\alpha-1}\exp(-S/\eta)$$

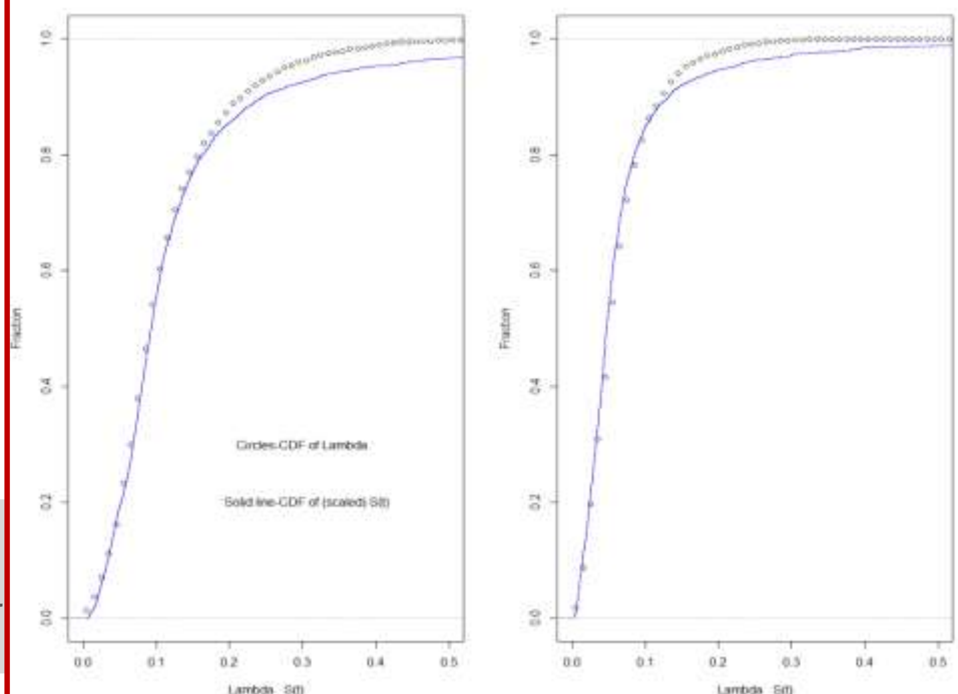
Scale parameter: $\eta = \beta/c$
and $c = \overline{\Lambda}/m_s$

Shape parameters are equal: α

All we need is an estimate of m_s!

Storage, S(t), sampled by assuming that $\Lambda(t)$ is the parameter of a linear reservoir.

$$S(t) = \frac{Q(t)}{1 - e^{-\Lambda(t)}}$$



The average recession $\bar{\Lambda}$, represents a subsurface state of mean storage (m_S)

- Unit hydrograph in a state of mean storage:

$$u_{\bar{\Lambda}}(t) = \bar{\Lambda} e^{-\bar{\Lambda}(t-t_0)}$$

- Weights distributing impulse in a state of mean storage:

$$\mu(\bar{\Lambda})_j = \int_{(j-1)\Delta t}^{(j)\Delta t} u_{\bar{\Lambda}}(t)dt \quad j = 1..J, \quad \sum \mu(\bar{\Lambda})_j = 1$$

- J is the tempopral scale of $u_{\bar{\Lambda}}(t)$

Steady-state mean annual runoff represents mean storage (m_S)

Excess water input X necessary to maintain mean annual runoff (MAR)

$$X[mm/day] = (1000 * MAR[m^3/s] * 86400[s]) / A[m^2]$$

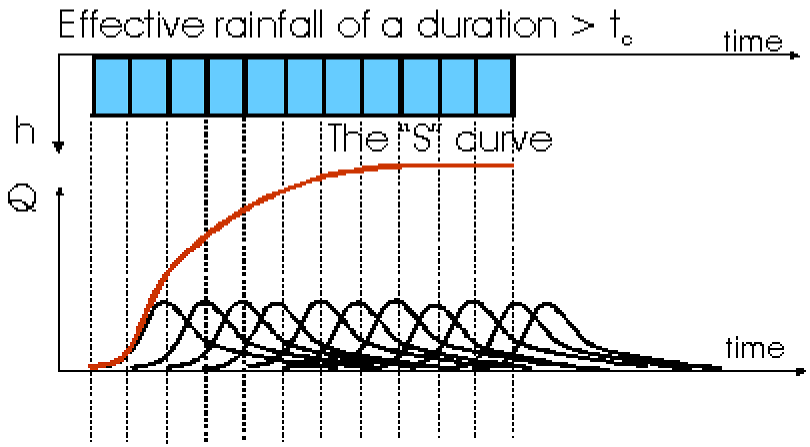


Figure 4.4. The construction of the "S" curve

What is left in the soils (S_S) at steady-state?

- The S curve is obtained after J days of input X. Balancing the volumes gives us : $J \cdot X = S_S + Q_S$

where:

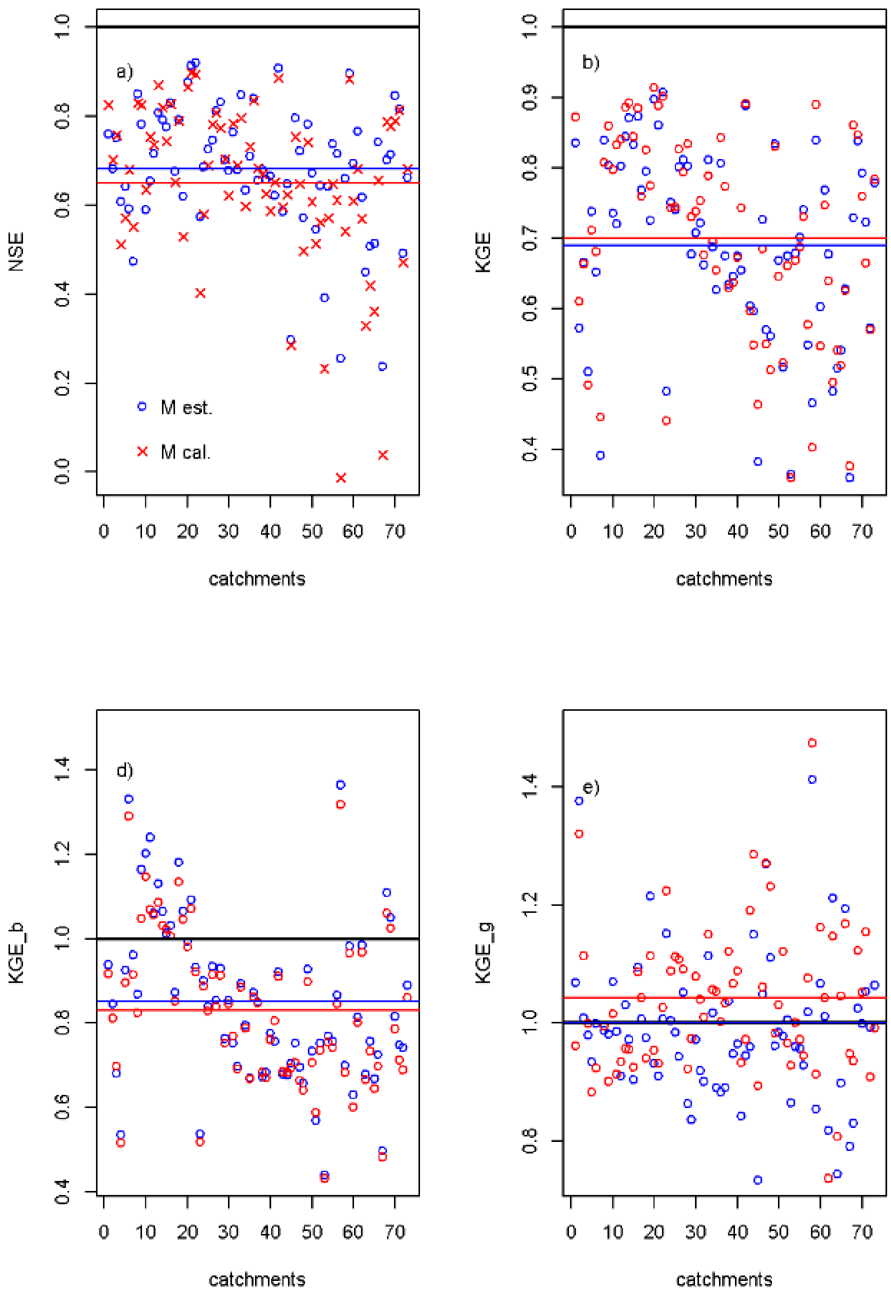
$Q_S = \sum_{k=1}^{J} \sum_{j=1}^{k} X \cdot \mu(\overline{\Lambda})_j$ (runoff) and

$S_S = \sum_{k=1}^{J-1} \sum_{j=k+1}^{\mathrm{J}} X \cdot \mu(\overline{\Lambda})_j = m_S$ (storage)

Recall information needed: $\overline{\Lambda}$ and MAR

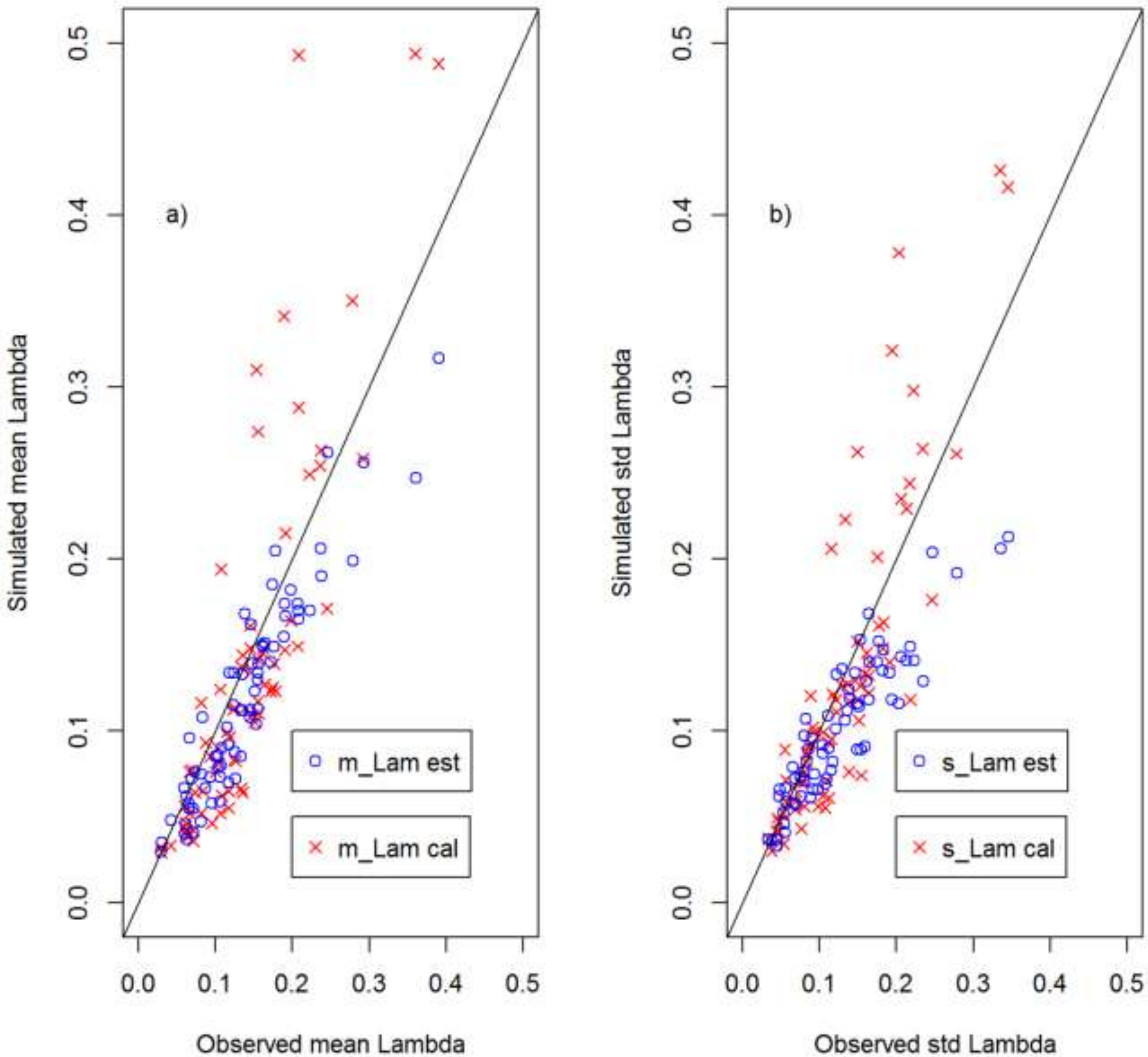
Results: New groundwater dynamics

- Comparing performances of DDD with θ_M (calibrated) and m_S (estimated)
- Tested for 73 catchments, with no corrections of meteorology



	NSE	KGE	KGE_r	KGE_b	KGE_g
DDD_m_S	0.68	0.69	0.83	0.85	1.0
DDD_θ_M	0.66	0.70	0.85	0.83	1.04

Mean and standard deviation of $\Lambda = log(Q(t)) - log(Q(t + \Delta t))$



	RMSE mean Λ	RMSE std Λ
DDD_m_S	0.275	0.389
DDD_θ_M	0.511	0.392

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Thank you for your attention!

References

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A rainfall-runoff model parameterized from GIS and runoff data

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Hydrol. Process. (2014)
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Use of a parsimonious rainfall–run-off model for predicting hydrological response in ungauged basins

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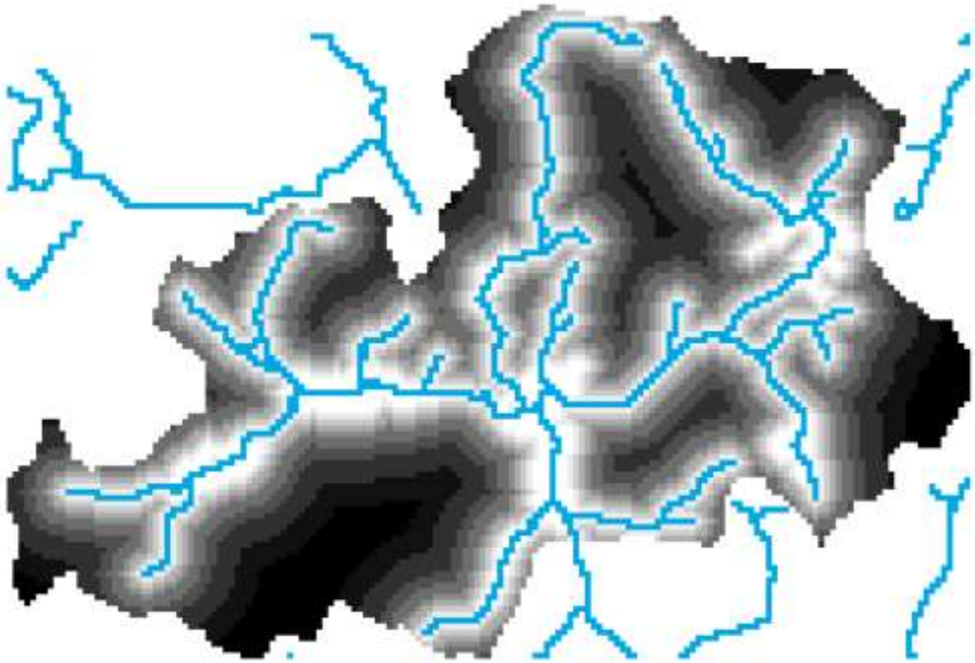
Distance distributions and runoff dynamics

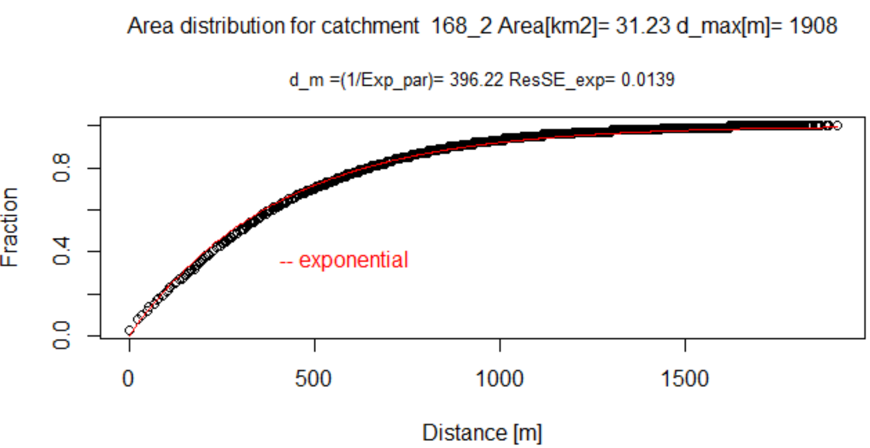
•Creating equidistant buffers around the river network (blue) is a way to determine the distribution of distances from a point in the catchment to its nearest river reach.

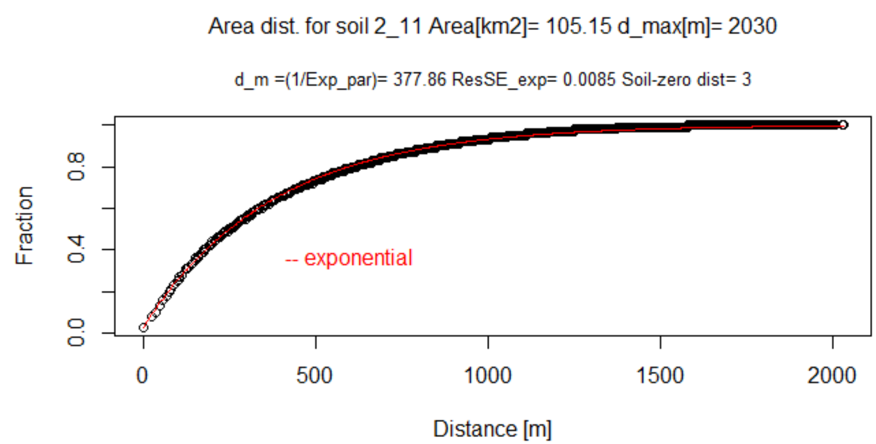


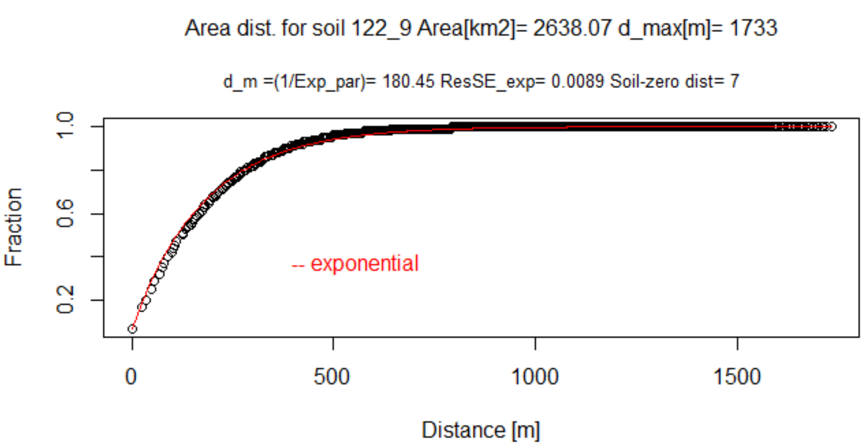
The distance distribution is exponential!

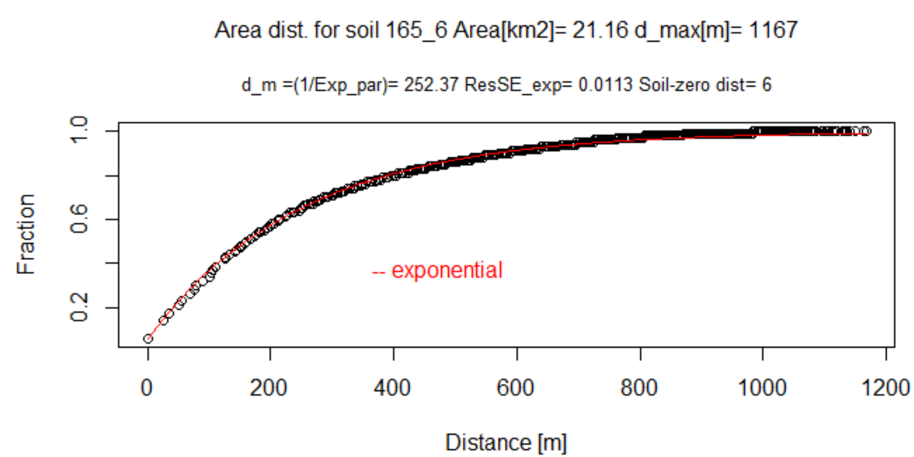
•For more than 120 catchments in Norway, the empirical DD is well approximated by an exponential distribution

$$f(d) = \gamma e^{-\gamma d}, \qquad \gamma = 1/\bar{d}$$



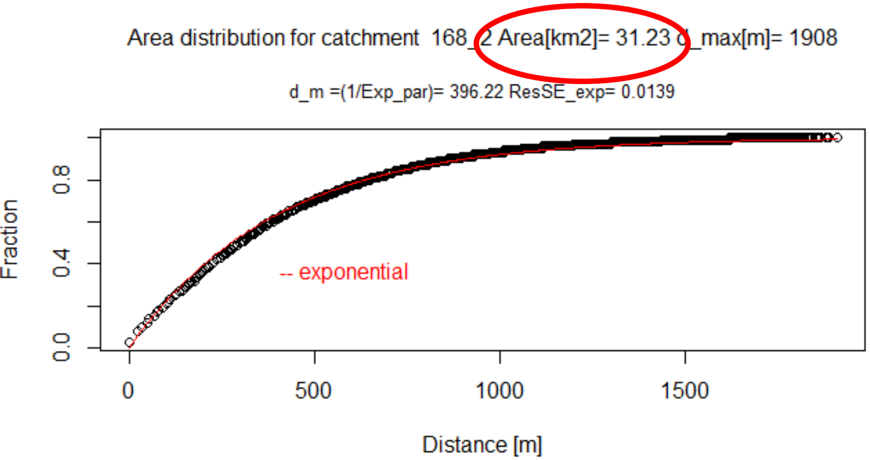


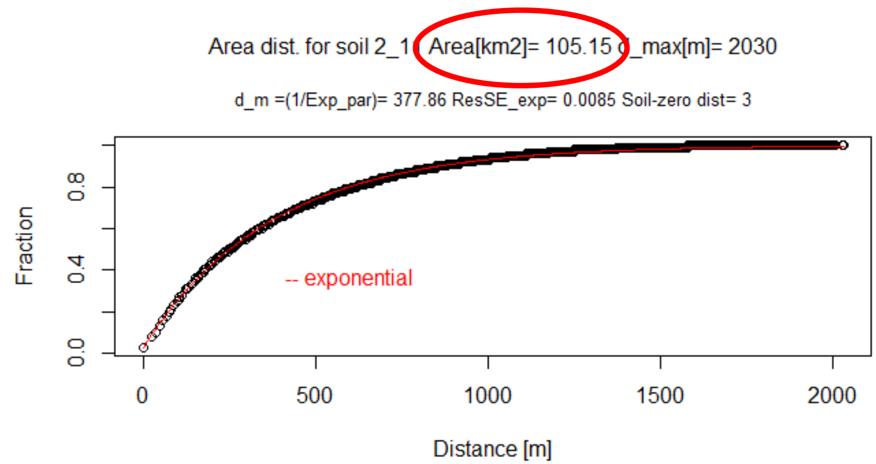


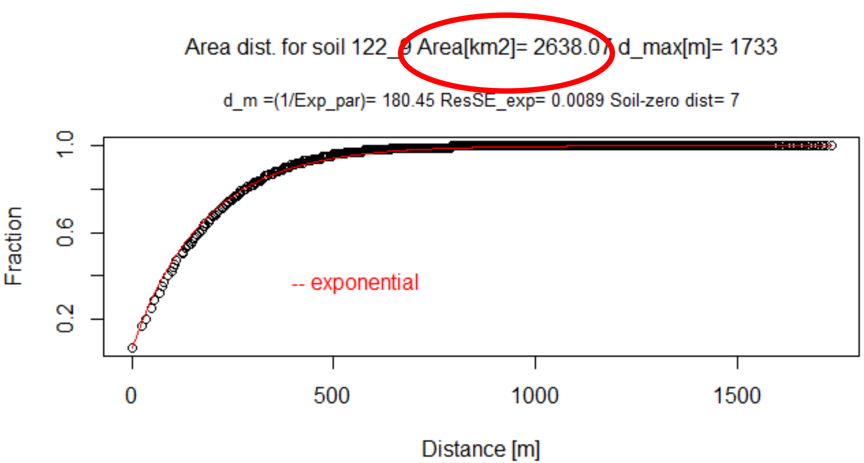


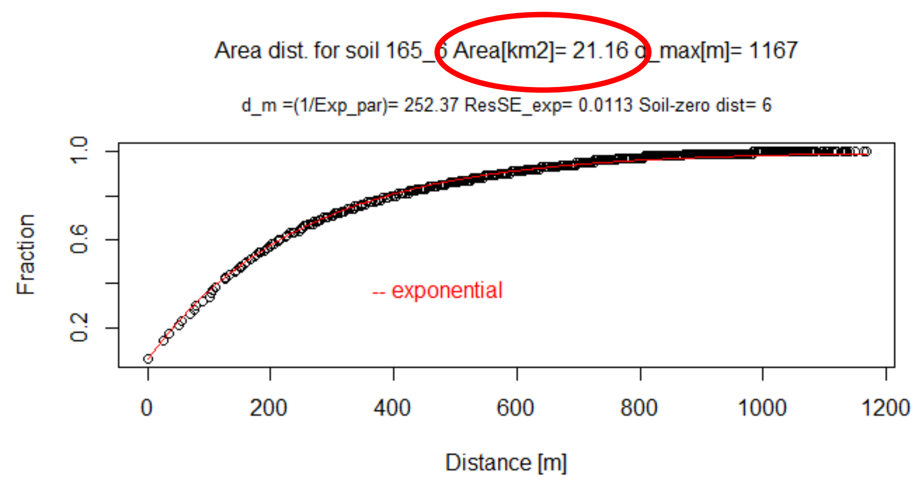
The distance distribution is exponential!

•For more than 120 monitored catchments in Norway, the empirical DD is well aproximated by an exponential distribution, big and small



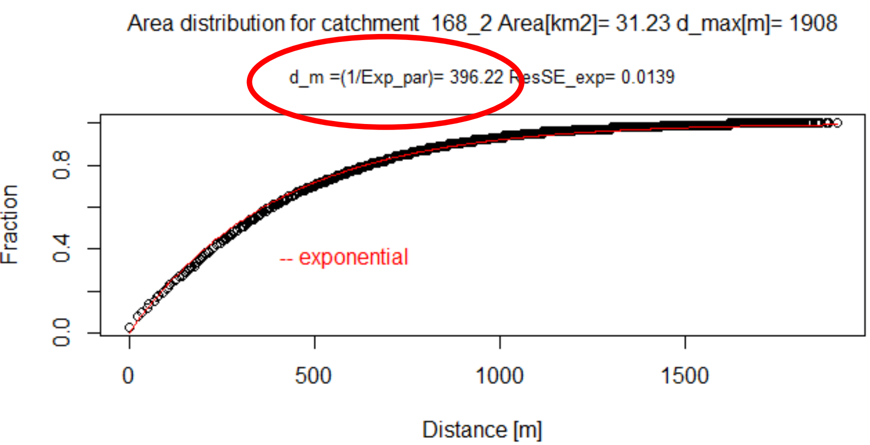


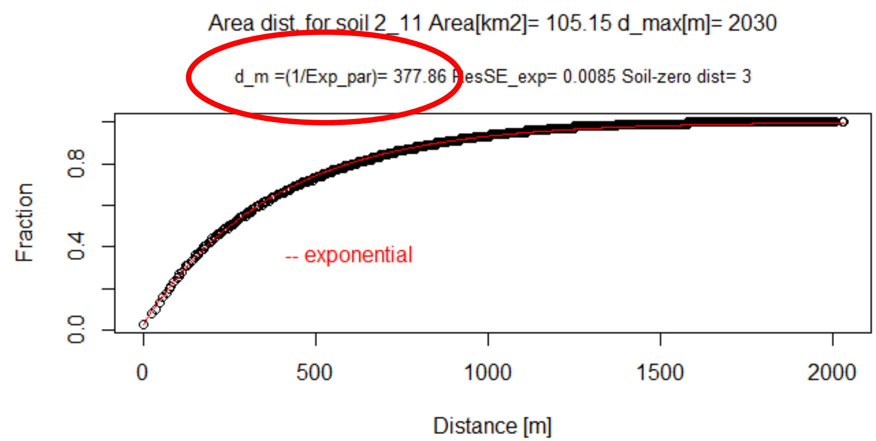


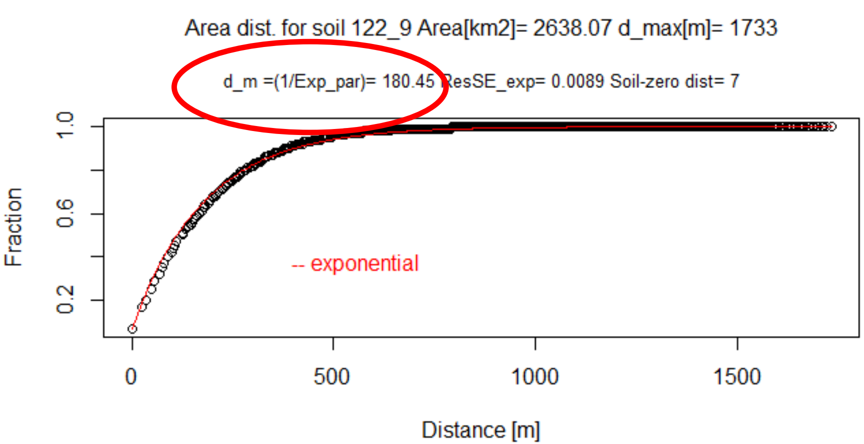


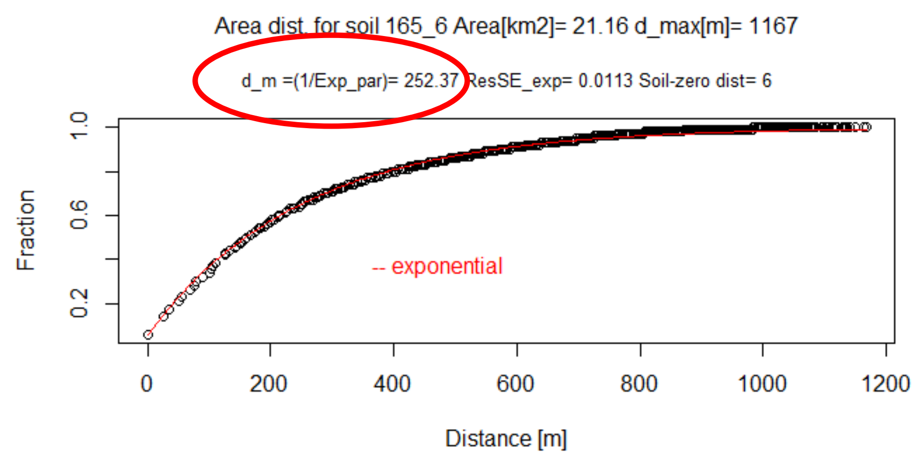
The distance distribution is exponential!

•For more than 120 monitored catchments in Norway, the empirical DD is well aproximated by an exponential distribution, but the parameter varies







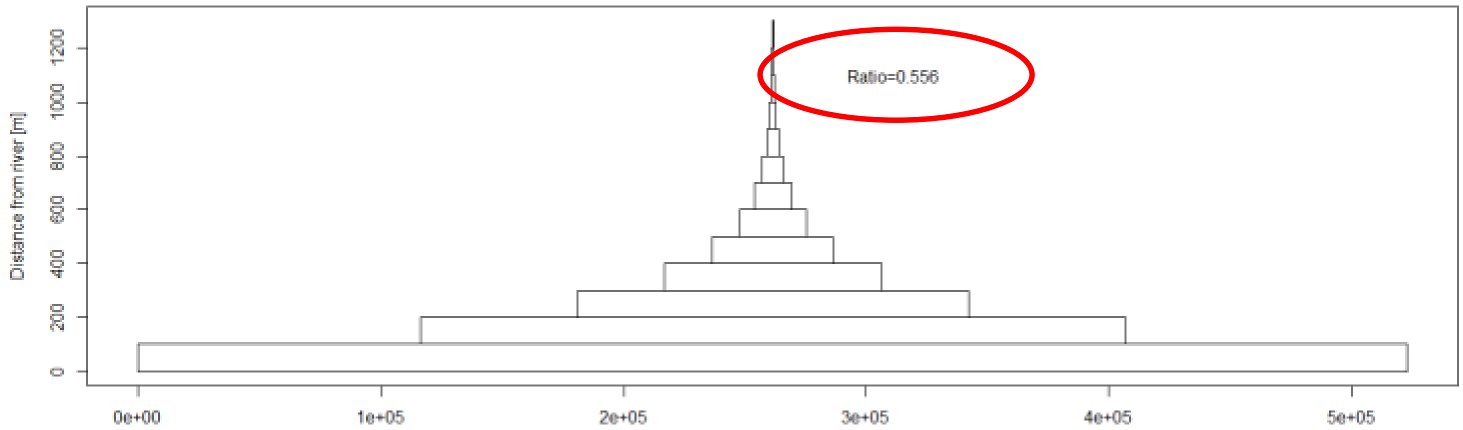


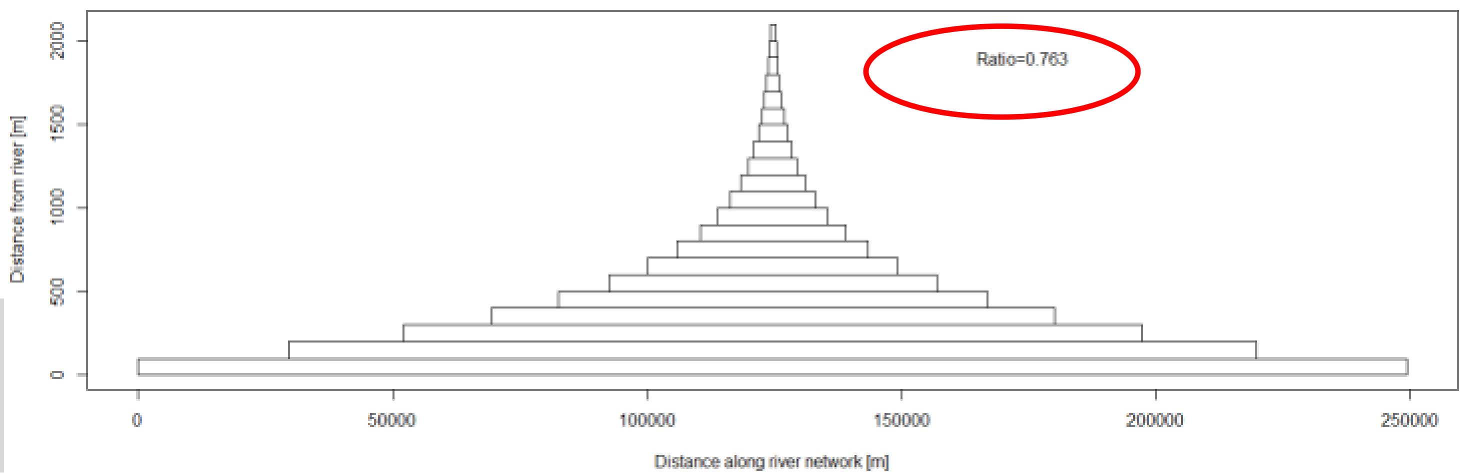
Another way to visualize the distance distribution..

- The consecutive areas for each Δd in the DD are plotted.

- The ratio κ between consecutive areas is constant (feaure of the exponential distribution)

- Is this the shape of the aquifer?



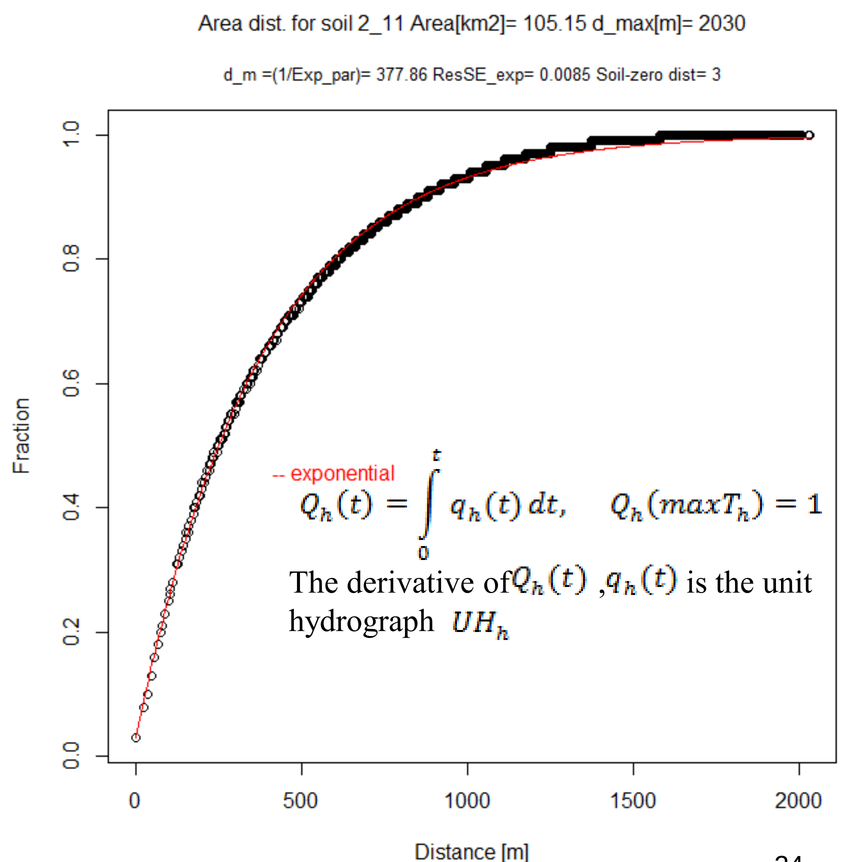


From Distance Distributions to Unit Hydrographs-assigning velocities

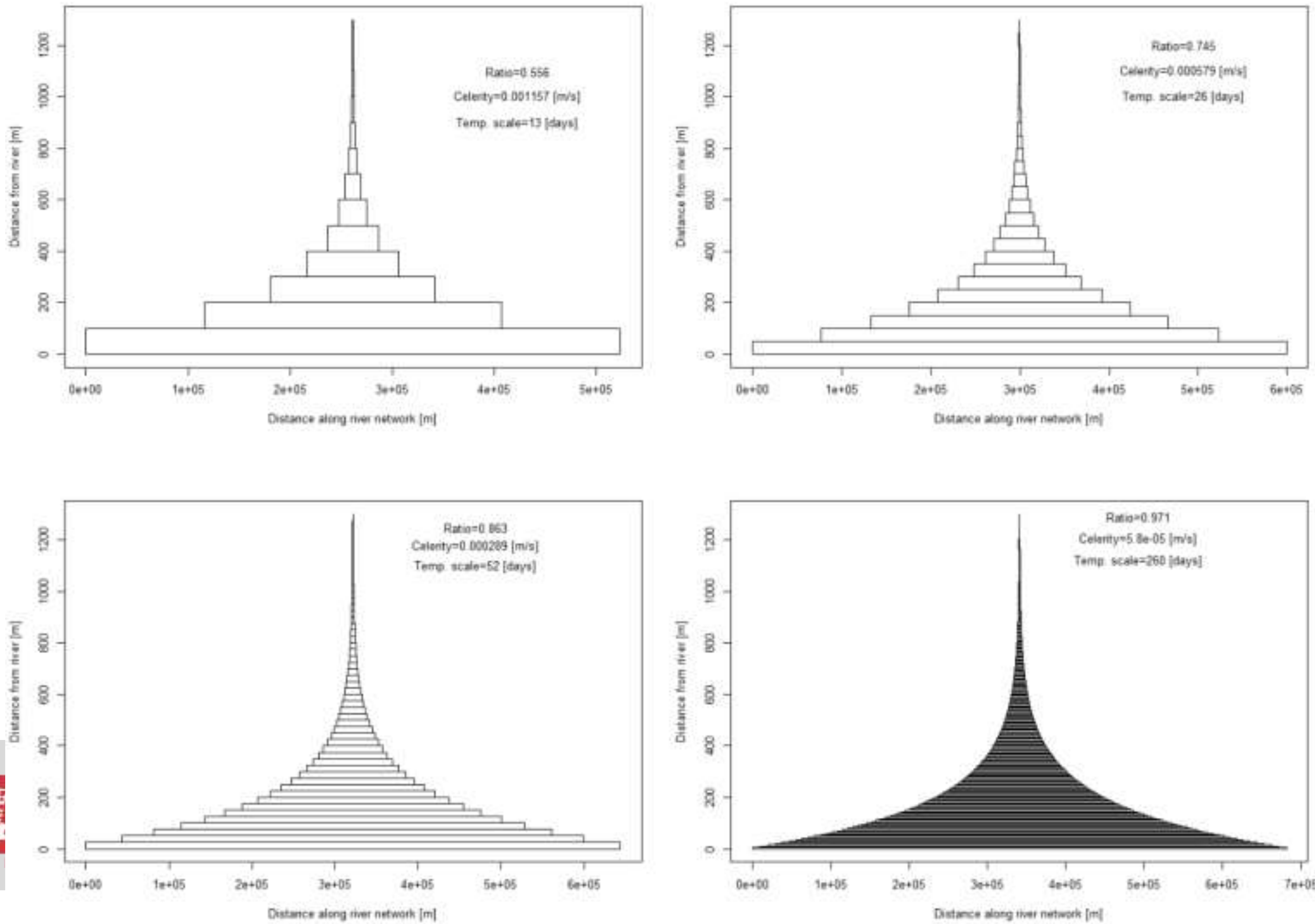
- If waves of water travel with a constant wave velocity v, (celerity), the exponential distribution of distances becomes an exponential distribution of travel times, i.e. the variable d is replaced with d/v with parameter

$$\xi = -\log(\kappa)/\Delta t$$

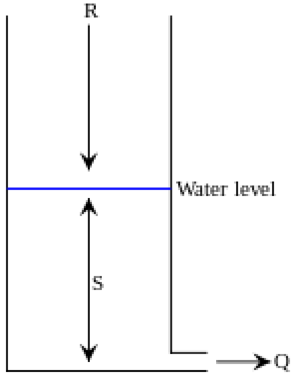
- A travel time distribution constitutes a Unit Hydrograph (UH).
- The UH distributes the input (rain/snowmelt) in time to the outlet where we observe it as runoff. (The UH is basically a set of weights)

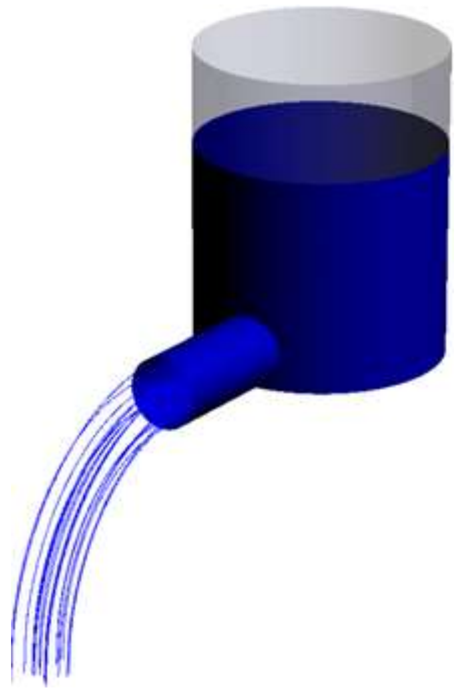


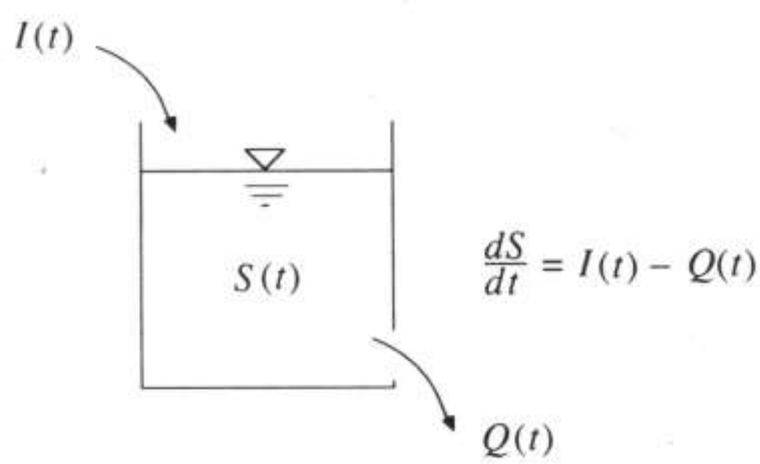
For one catchment: different celerities gives different temporal scales to the unit hydrographs, but the shape remains the same and uniqe to that catchment



How is a linear reservoir perceived?







Linear reservoirs and distance distributions

- The constancy (κ) of the ratio between consecutive areas (distance distribution) also holds for consecutive runoff volumes (travel time distribution) because of its exponential shape.
- The constancy is also feature of the (super famous) linear reservoir
- Hence, a linear reservoir with runoff coefficient φ;

$Q(t) = \varphi S(t)$, where $S(t)$ is the reservoir (storage) can also be expressed as

$Q(t) = (1 - \kappa)S(t)$ where κ can be expressed in terms of the parameter of the exponential travel time distribution

$$\xi = -\log(\kappa)/\Delta t$$

- **The linear reservoir model is a result of exponential distance distributions**