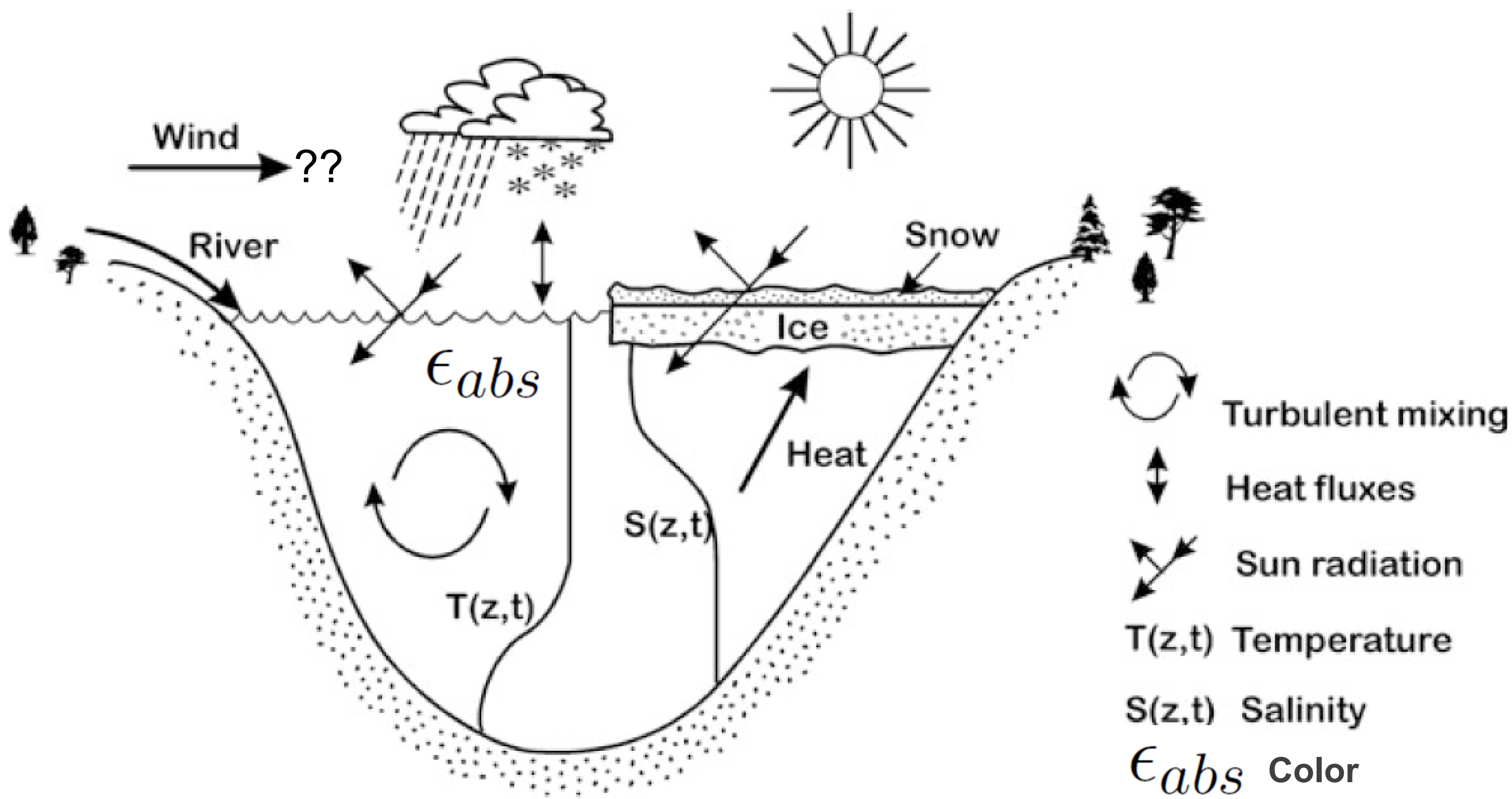
Norwegian University
of Life Sciences

Innsjøsirkulasjon og global oppvarming

Norsk hydrologiråds konferanse på NMBU, 6.mai 2019, om:
Klimaendringer og vannkvalitet- hva kan vi forvente oss?

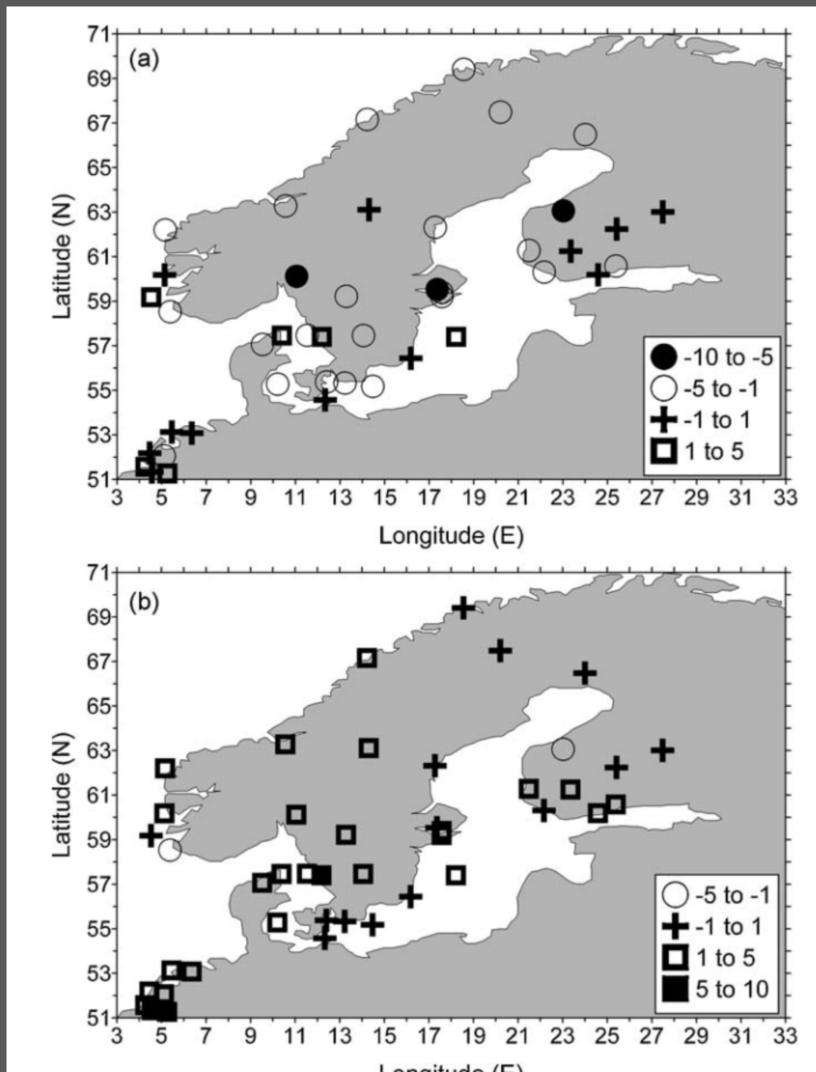
Sverre Anmarkrud, Alemayehu Adugna Arara, Anne Kværnø, and Nils-Otto Kitterød

Lake temperature



Omstedt (2015): Guide to Process Based Modeling of Lakes and Coastal Seas

Wind velocities and global warming?



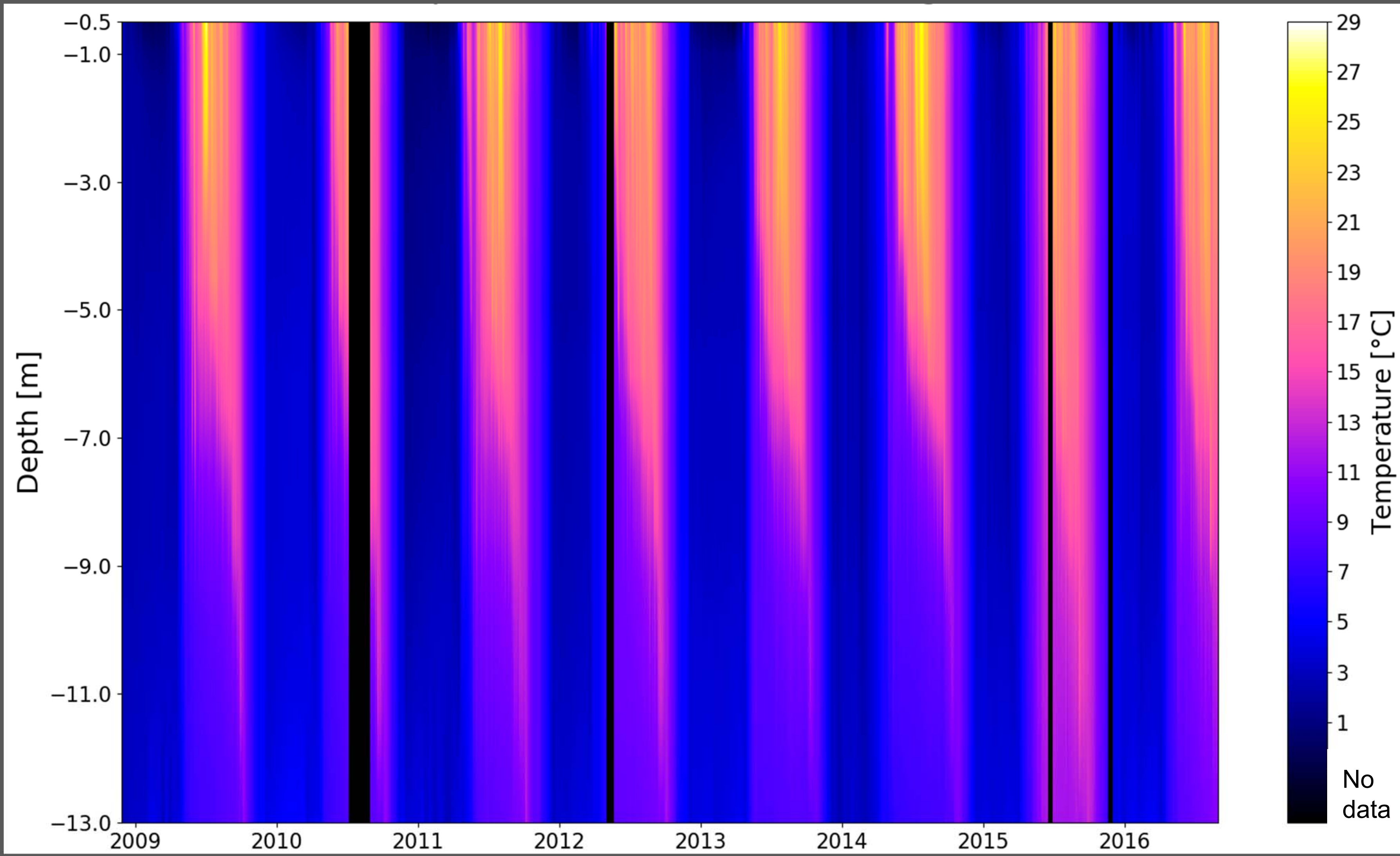
$$E = \frac{1}{2}\rho U^3$$

E = energy density (W m^{-2}),
ρ = air density (kg m^{-3}),
U = wind speed at hub-height (m s^{-1}).

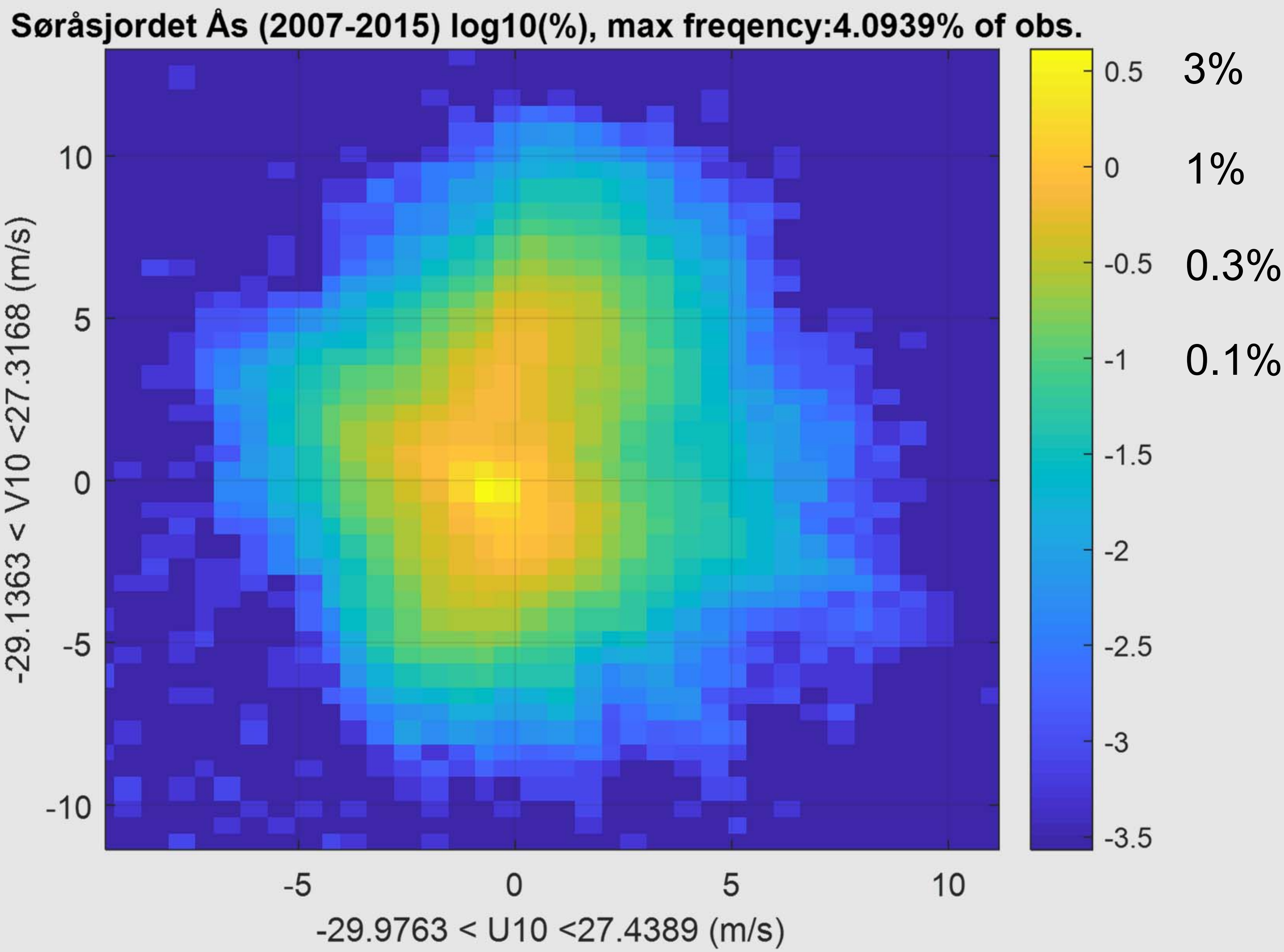
Ensemble average difference in percent of (a) energy density and (b) 50-year return period wind speed computed using a probabilistic empirical downscaling Approach for 43 stations across northern Europe based on output from 8-GCMs (BCCR-BCM2.0, CGCM3.1, CNRM-CM3, ECHAM5/MPI-OM, GFDL-CM2.0, GISS-ModelE20/Russell, IPSL-CM4, and MRI-CGCM2.3.2.). The future time period is 2081–2100, while the historical period in 1961–1990. **A positive value indicates higher energy density or extreme wind in the later time period**

S.C. Pryor, R.J. Barthelmie / Renewable and Sustainable Energy Reviews 14 (2010) 430–43

Temperature observations from lake Årungen



Probability density function of wind velocities from Søråsjordet in the period from 2007 to 2015, 10 min average.



The Simstrat Model

https://www.eawag.ch/en/department/surf/projects/simstrat/

- 1D physical lake model.
- For simulation of stratification and mixing in deep lakes.
- Originally developed by Goudsmit et al. (2002).
- Applied to lakes with different physical properties.
- Open source: The Simstrat code is freely available on Github:
  - https://github.com/Eawag-AppliedSystemAnalysis/Simstrat/

The Main properties of the Simstrat Model

https://www.eawag.ch/en/department/surf/projects/simstrat/

- Simulates temperature, salinity, stratification and ice cover.
- k-ε model for turbulent mixing and buoyancy oscillations.
- Includes energy transfer to mixing via internal seiches
- Hydrology: inflows can be added at specified depths or with density-dependent intrusions.
- Variable lake surface levels.
- Programmed in object-oriented Fortran 2003.
- Parameter estimation using PEST is implemented

Basic Model Equations

The basic set of equations for temperature T, mean horizontal velocity components u and v with respect to x and y, turbulent kinetic energy (TKE) per unit mass k and the TKE dissipation rate ϵ are

$$\frac{\partial T}{\partial t} = \frac{1}{A}\frac{\partial}{\partial z}\left(A(\nu_t' + \nu')\frac{\partial T}{\partial z}\right) + \frac{1}{\rho_0 c_p}\frac{\partial H_{sol}}{\partial z} + \frac{dA}{dz}\frac{H_{geo}}{A\rho_0 c_p}, \qquad \nu_t' = c_\mu' \frac{k^2}{\epsilon}, \tag{1}$$

$$\frac{\partial u}{\partial t} = \frac{1}{A}\frac{\partial}{\partial z}\left(A(\nu_t + \nu)\frac{\partial u}{\partial z}\right) + fv, \qquad \nu_t = c_\mu \frac{k^2}{\epsilon}, \tag{2}$$

$$\frac{\partial v}{\partial t} = \frac{1}{A}\frac{\partial}{\partial z}\left(A(\nu_t + \nu)\frac{\partial v}{\partial z}\right) - fu, \tag{3}$$

$$\frac{\partial k}{\partial t} = \frac{1}{A}\frac{\partial}{\partial z}\left(A\nu_k\frac{\partial k}{\partial z}\right) + P + P_{seiche} + B - \epsilon, \qquad \nu_k = \frac{c_\mu}{\sigma_k}\frac{k^2}{\epsilon}, \tag{4}$$

$$\frac{\partial \epsilon}{\partial t} = \frac{1}{A}\frac{\partial}{\partial z}\left(A\nu_\epsilon\frac{\partial \epsilon}{\partial z}\right) + \frac{\epsilon}{k}\left(c_{\epsilon 1}(P + P_{Seiche}) + c_{\epsilon 3}B - c_{\epsilon 2}\epsilon\right), \qquad \nu_\epsilon = \frac{c_\mu}{\sigma_\epsilon}\frac{k^2}{\epsilon}, \tag{5}$$

$$P = \nu_t\left(\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right), \qquad B = -\nu_t' N^2, \quad N^2 = -\frac{g}{\rho_0}\frac{\partial \rho}{\partial z}, \tag{6}$$

$$P_{Seiche}(z) = \frac{1 - 10\sqrt{C_{Deff}}}{\rho_0 c A_{boundary}} N^{2q}\frac{dA}{dz}\gamma E_{seiche}^{3/2}, \quad E(t) = \left(\frac{1}{E(t=0)} + \frac{\gamma}{2}t\right)^{-2}, \tag{7}$$

where

- ρ_0 is reference density,
- c_p is specific heat of lake water,
- A is the surface area of the lake at depth z and $A_{boundary}$ is the total bottom area,
- $H_{sol} = H_{Ssol,0}(1 - r_s)e^{-\epsilon_{abs} z}$ is the shortwave solar radiation penetrating the water, $H_{Ssol,0}$ is measured solar radiation above the water surface, r_s and ϵ_{abs} are the reflection and extinction coefficient of the lake water
- H_{geo} is the geothermal heat flux,
- f is the Coriolis parameter,
- ν and ν_t are molecular and turbulent viscosity (momentum),
- ν_t' and ν' are molecular and turbulent diffusivity of temperature,
- ν_ϵ and ν_k are the turbulent diffusivities of energy dissipation and TKE,
- P is the shear stress production,
- B is the buoyancy production and N is the Brunt-Väisälä frequency,
- P_{Seiche} is the production of TKE due to internal seiching,
- $c_{\epsilon 1}, c_{\epsilon 2}, c_{\epsilon 3}, c_\mu, c_\mu', \sigma_k$ and σ_ϵ are model constants,
- γ is a constant of proportionality (can be estimated),
- C_{Deff} is effective bottom friction coefficient and
- E_{Seiche} is total energy contained in the Seiche motion.

The stochastic wind model

3.1 The model

Let $Z(t) = [u(v), v(t)]$ be the vector representing the wind in two orthogonal directions (east-west and south-north). A stationary and two-points autocorrelated process can be defined by the following stochastic linear differential equations:

$$dZ = AZdt + BdW \tag{20}$$

where A and B are 2×2 matrices and W is a two-dimensional Brownian process. The exact solution of this SDE is given by (Arnold, 1974)

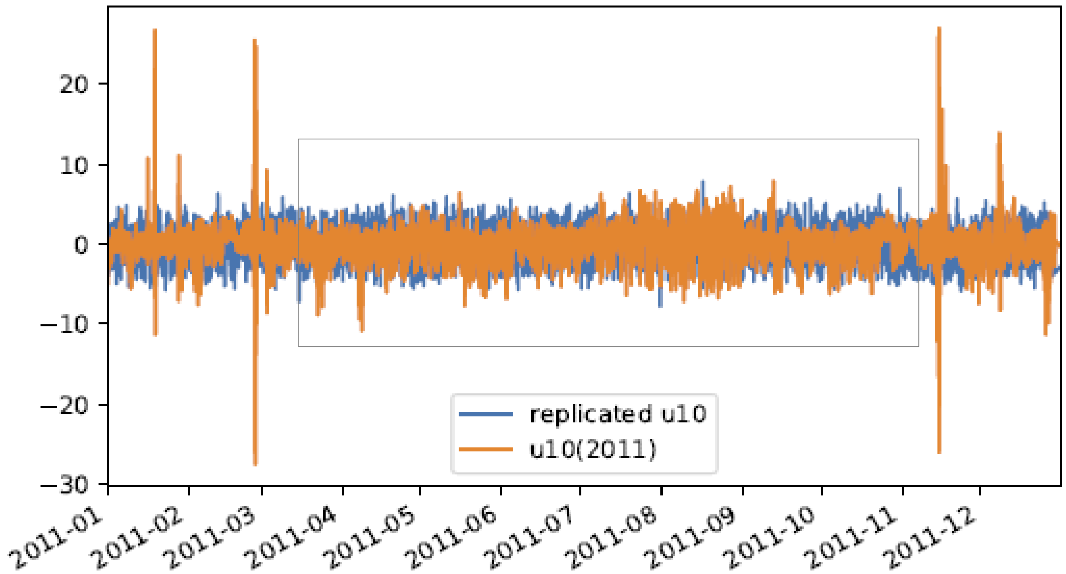
In this manuscript, we would calibrate the model to fit with measured data, and compare the effect of using synthetic datas compared to measured data for the temperature profile computed by Simstrat. In the long run, we would like test different wind scenarios, in which case the drift and diffusion terms becomes time-dependent, that is

$$dZ = A(t)Zdt + B(t)dW \tag{24}$$

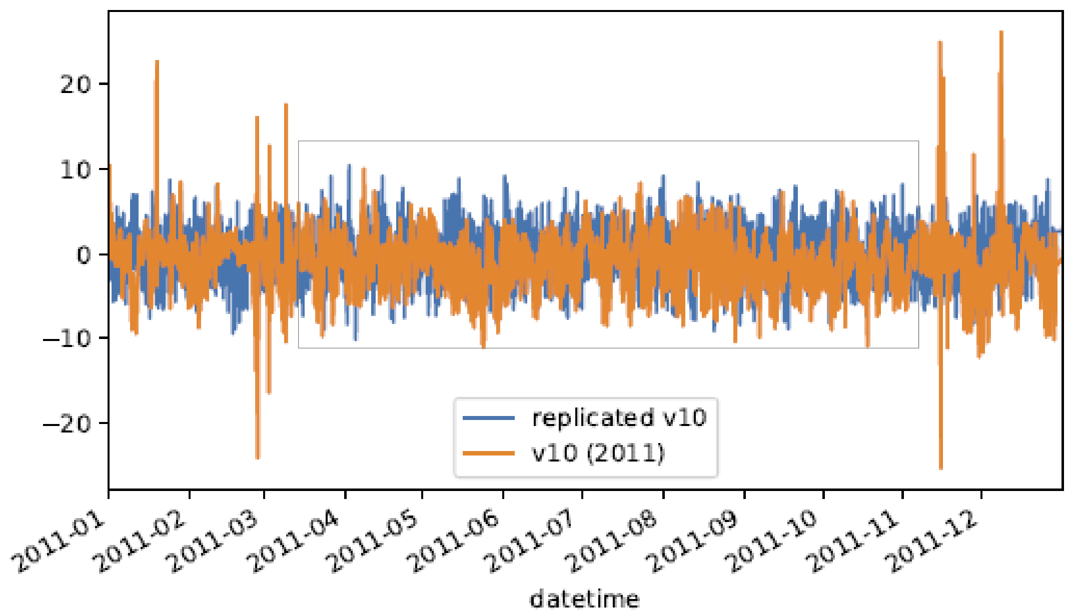
This can be solved with the the well known Euler-Maruyama method (Kloeden and Platen, 1992), which for (24)

$$Z_{i+1} = Z_i + hA(t_i)Z_i + B(t_i)\sqrt{h}\xi_i, \tag{25}$$

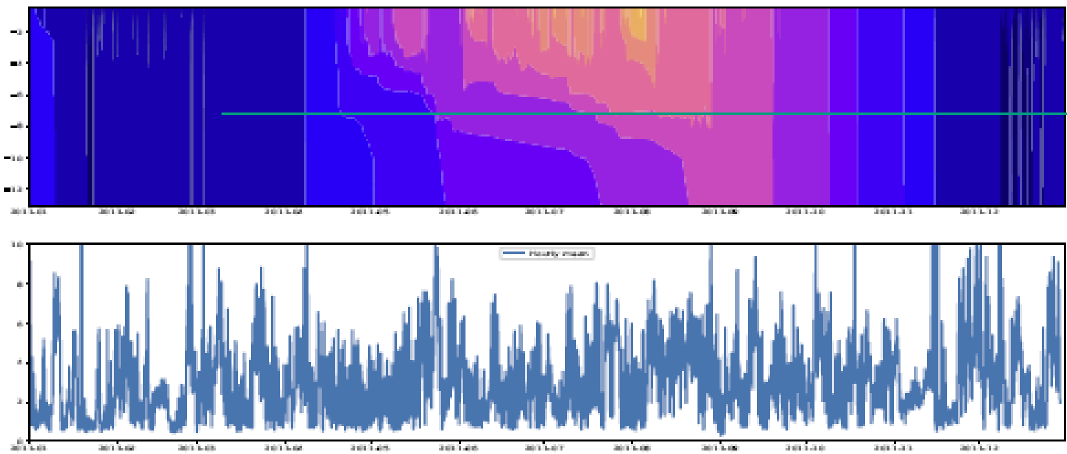
where $\xi_i \sim \mathcal{N}(0, I_2)$.



East-West

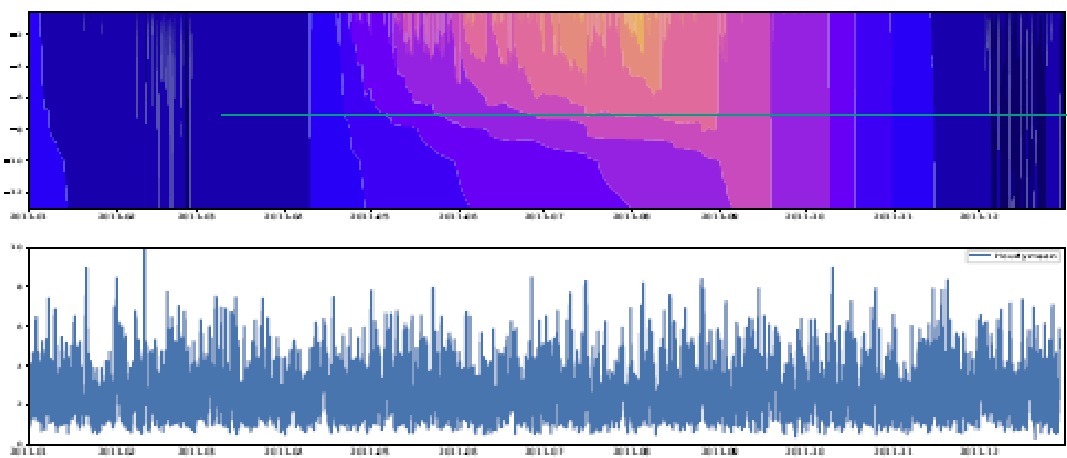


North-South



(a) Temperature profile using measured wind data

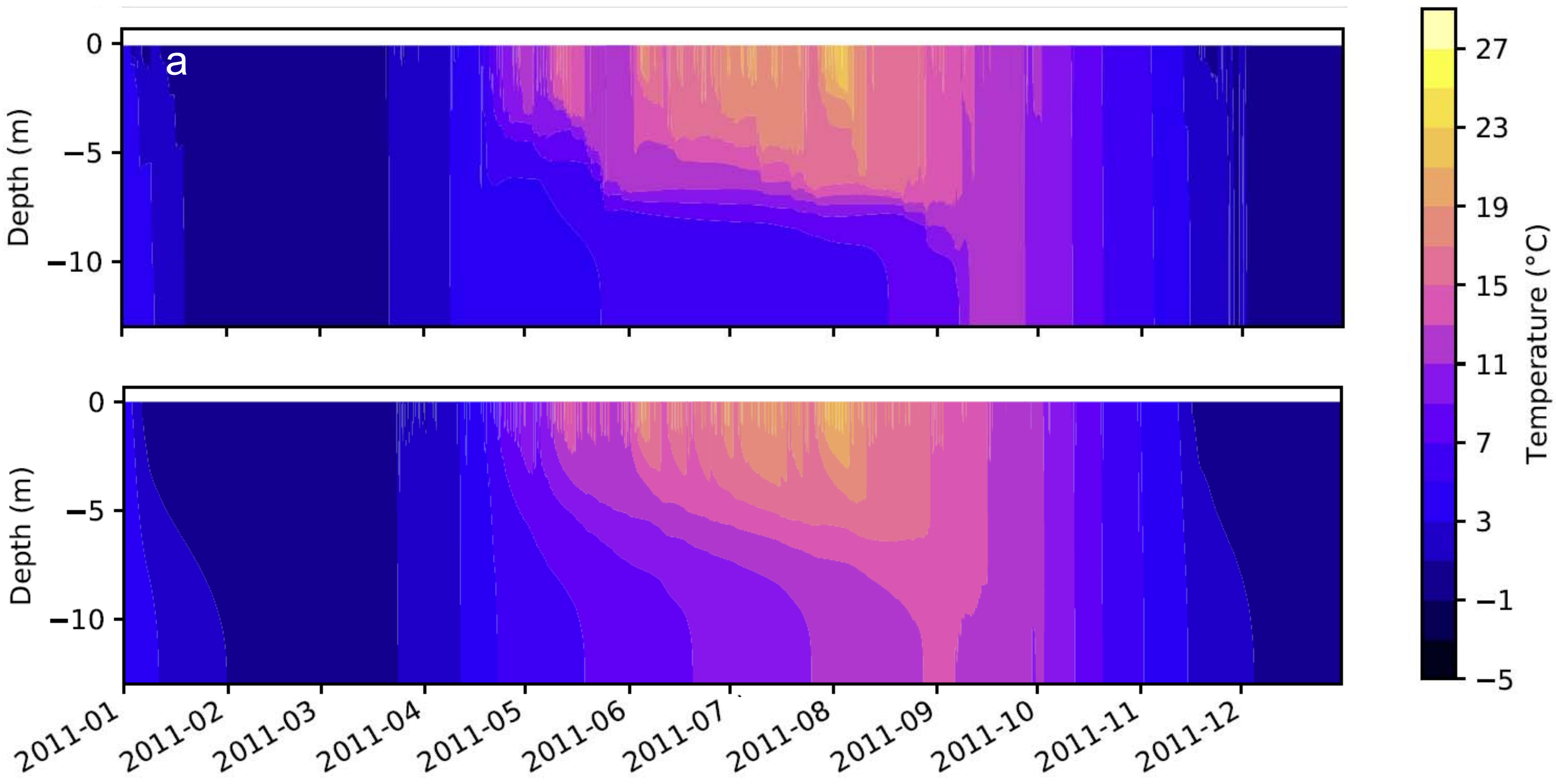
Observed wind

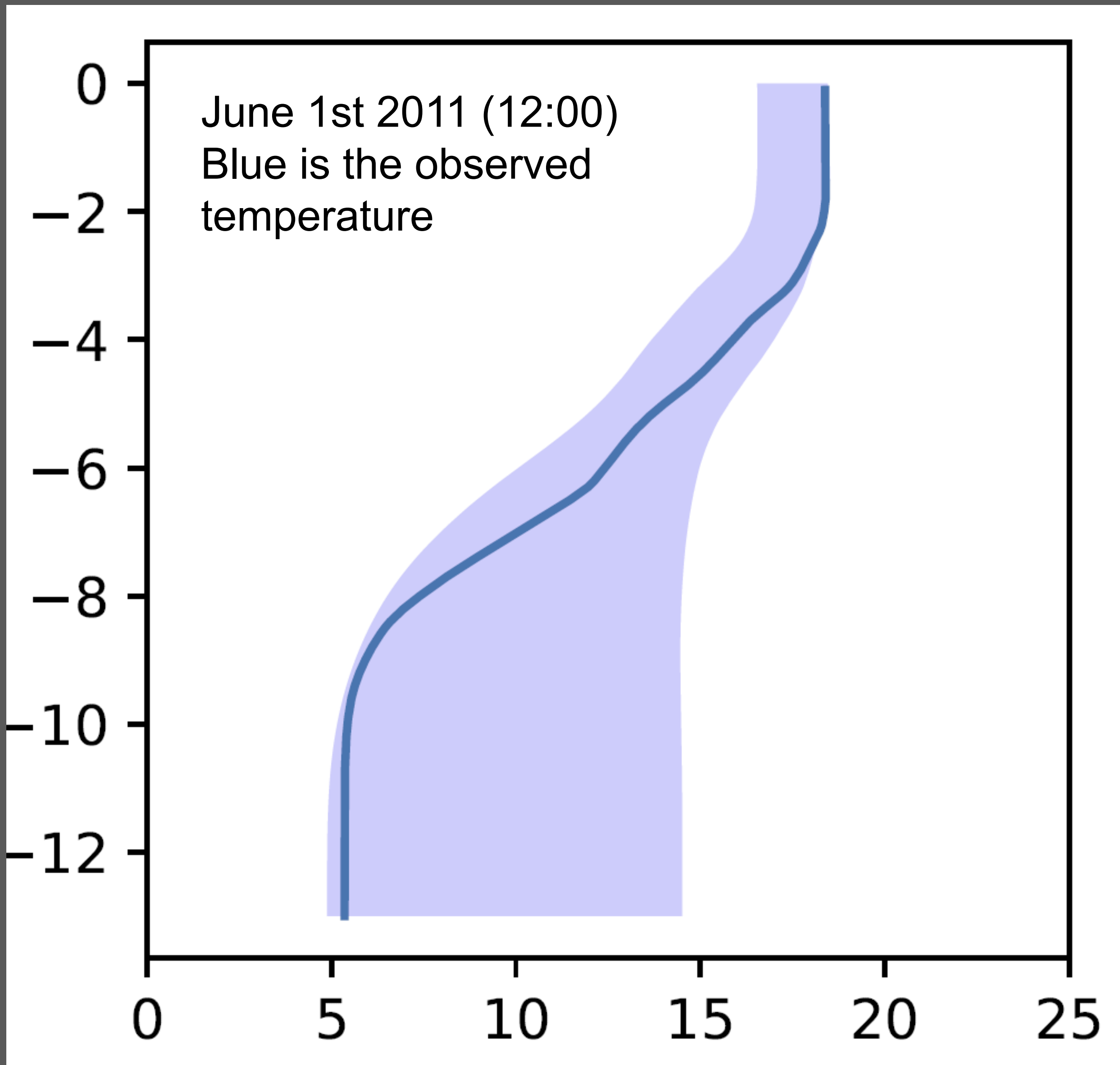


(b) Temperature profile using one simulated wind field

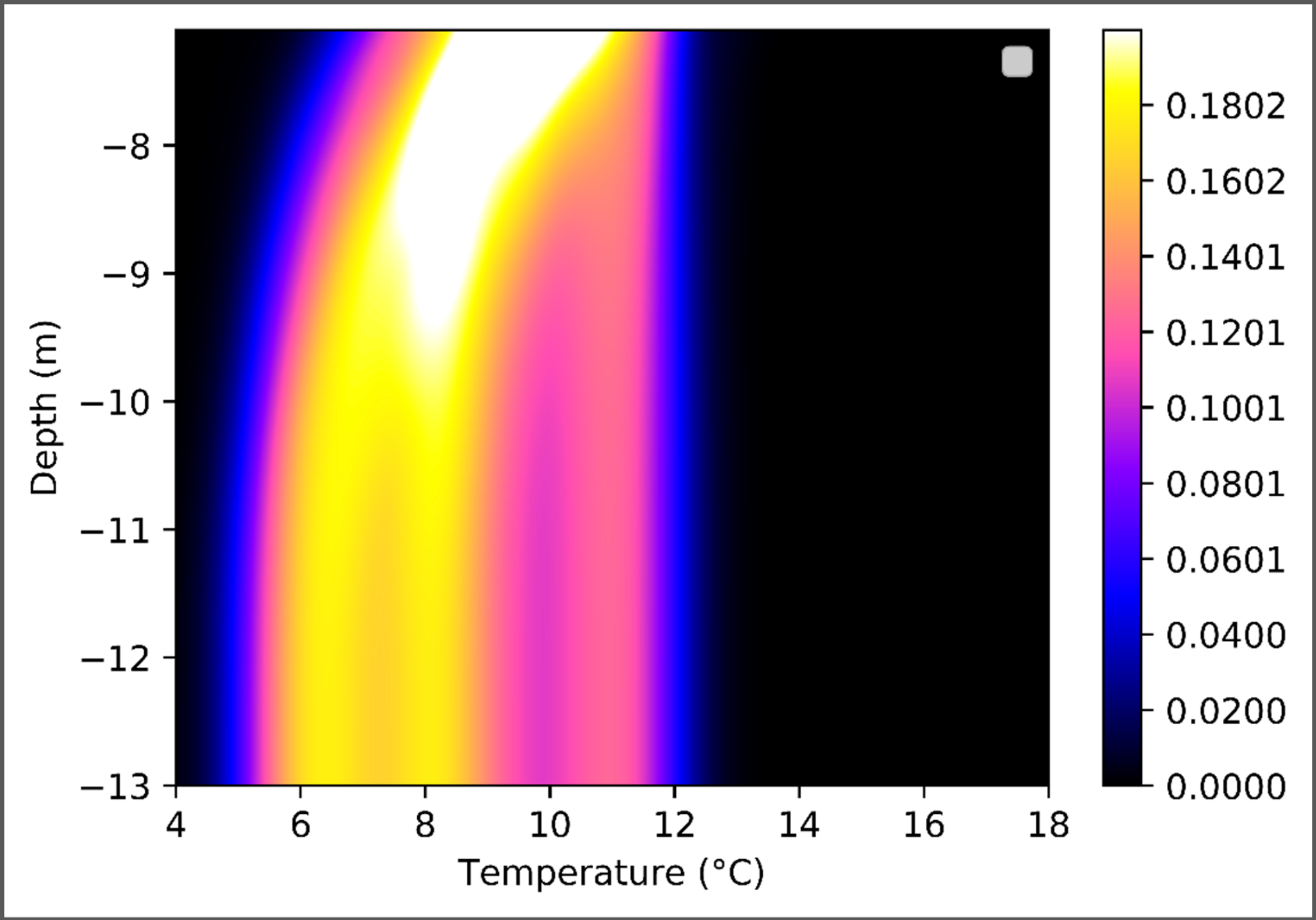
Synthetic wind

Temperature simulations




June 1st 2011 (12:00)
Blue is the observed
temperature
0
−2
−4
−6
−8
−10
−12
0
5
10
15
20
25

Probability density function of temperature (T) given depth (z): f(T|z)



Questions for further work:

1) What's the impact of wind to temperature stratification in lakes?
2) Do we expect more circulation because of more wind or vice versa?
3) Do we need wind observations to mimic temperature in lakes, or can a stochastic wind model do the job?
4) Is a simple stochastic model sufficient (mean and covariance), or do we need to include more physics (e.g. coupling of temperature)?
5) What time resolution is necessary? Challenge: Mixing parameters depends on variance in wind velocities.